Problem 1.

P	Q	$P \Rightarrow Q$	P or Q	$(P$ or $Q) \iff Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	T

Since the two blue columns coincide, the statements are equivalent.

Problem 2. Suppose that $a \mid b$ and $a \mid c$. Then there exist integers q and r such that $b = q \cdot a$ and $c = r \cdot a$. Therefore, $b + c = q \cdot a + r \cdot a = (q + r) \cdot a$, so since $q + r$ is an integer, we see that $a \mid b + c$.

Problem 3.

(i) Since squares of real numbers are nonnegative we know that $(x - y)^2 \geq 0$. Expanding yields $x^2 - 2xy + y^2 \geq 0$ or equivalently $x^2 + y^2 \geq 2xy$.

(ii) Expanding both sides we have to show that $a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2 \geq a^2c^2 + 2abcd + b^2d^2$, or equivalently $a^2d^2 + b^2c^2 \geq 2abcd$. Applying part (i) to $x = ad$ and $y = bc$ we find $a^2d^2 + b^2c^2 = x^2 + y^2 \geq 2xy = 2abcd$, as desired.

Problem 4. Let $P(n)$ be the statement "$3^n - 1$ is even". We prove by induction that $P(n)$ is true for all integers $n \geq 1$.
Induction basis. Note that $3^1 - 1 = 2$ is even, so $P(1)$ is true.
Induction hypothesis. Suppose that $P(k)$ is true for some integer $k \geq 1$, so $3^k - 1$ is even.
Induction step. Since $3^k - 1$ is even there exists an integer q with $3^k - 1 = 2q$. Now

$$3^{k+1} - 1 = 3 \cdot 3^k - 1 = 3 \cdot (2q + 1) - 1 = 6q + 2 = 2 \cdot (3q + 1),$$

so since $3q + 1$ is an integer, $3^{k+1} - 1$ is even, i.e. $P(k + 1)$ is true. By induction we are now done.

Problem 5. To show this equality of sets we have to show two inclusions, namely $A \Delta B \subseteq (A \cup B) - (A \cap B)$ and $A \Delta B \supseteq (A \cup B) - (A \cap B)$.

$\subseteq$ Let $x \in A \Delta B$. Then $x \in A - B$ or $x \in B - A$. We distinguish two cases.
- Suppose $x \in A - B$. Then $x \in A$, hence $x \in A \cup B$. Furthermore, $x \notin B$, so $x \notin A \cap B$. Therefore, $x \in (A \cup B) - (A \cap B)$.
- If $x \in B - A$ we can do the same argument with the roles of A and B reversed.

$\supseteq$ Let $x \in (A \cup B) - (A \cap B)$. Then $x \in A \cup B$, but $x \notin A \cap B$. Since $x \in A \cup B$, we know that either (1) x belongs to A but not to B, (2) x belongs to B but not to A or (3) x belongs to both A and B. However, since $x \notin A \cap B$, case (3) cannot occur. Again we consider 2 cases.
(1) In this case $x \in A$ and $x \notin B$, hence by definition $x \in A - B$. Therefore, we also have $x \in (A - B) \cup (B - A)$.
(2) This case is analogous: we find $x \in B - A$, hence also $x \in (A - B) \cup (B - A)$.

Problem 6. Let $P(n)$ be the statement "$u_n < 2^n$". We use strong induction to prove that $P(n)$ is true for all positive integers n.
Induction basis. Since $u_1 = 1 < 2^1$, $u_2 = 1 < 2^2$ and $u_3 = 1 < 2^3$, $P(1)$, $P(2)$ and $P(3)$ are all true.
Induction hypothesis. Suppose that for some integer $k \geq 1$ all three of $P(k)$, $P(k + 1)$ and $P(k + 2)$ are true, so $u_k < 2^k$, $u_{k+1} < 2^{k+1}$ and $u_{k+2} < 2^{k+2}$.
Induction step. We can now deduce that

$$u_{k+3} = u_{k+2} + u_{k+1} + u_k < 2^{k+2} + 2^{k+1} + 2^k = 4 \cdot 2^k + 2 \cdot 2^k + 2^k = 7 \cdot 2^k < 8 \cdot 2^k = 2^{k+3},$$

so $P(k+3)$ is true (For the second to last step we use $2^k > 0$, such that we can multiply the inequality $7 < 8$ by this number without changing the sign). By strong induction $P(n)$ is true for all integers n.